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On the D-wave state component of the deuteron in the Nambu-Jona-Lasinio model of light nuclei

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Abstract. The D-wave state component of the neutron-proton bound state in the deuteron is calculated in the Nambu-Jona-Lasinio model of light nuclei—the relativistically covariant quantum field-theoretic approach to the description of low-energy nuclear forces. The theoretical value of the fraction of the D-wave state relative to the S-wave state is equal to $\eta_{\rm d} = 0.0238$. This agrees well with the phenomenological value $\eta_{\rm d} = 0.0256 \pm 0.0004$ quoted by Kamionkowski and Bahcall (Astrophys. J. **420**, 884 (1994)).

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1 Introduction

The Nambu-Jona-Lasinio model of light nuclei or differently the nuclear Nambu-Jona-Lasinio (NNJL) model suggested in [1–3] represents a relativistically covariant quantum field-theoretic approach to the description of low-energy properties and interactions of the deuteron and light nuclei. The NNJL model is fully motivated by QCD [1]. The deuteron appears *in the nuclear phase of QCD* as a neutron-proton collective excitation, the Cooper np-pair, induced by a phenomenological local four-nucleon interaction. The NNJL model describes low-energy nuclear forces in terms of one-nucleon loop exchanges providing a minimal transfer of nucleon flavours from initial to final nuclear states and accounting for contributions of nucleon-loop anomalies which are completely determined by one-nucleon loop diagrams. The dominance of contributions of nucleon-loop anomalies to effective Lagrangians of low-energy nuclear interactions is justified in the large $N_{\rm C}$ expansion, where $N_{\rm C}$ is the number of quark colours.

Nowadays there is a consensus concerning the existence of non-nucleonic degrees of freedom in nuclei [4]. The non-nucleonic degrees of freedom can be described either within QCD in terms of quarks and gluons [5] or in terms of mesons and nucleon resonances [6]. In the NNJL model the non-nucleonic degrees of freedom of nuclei have been investigated in terms of the $\Delta(1232)$ resonance and calculated the contribution of the $\Delta\Delta$ component to the deuteron [2]. The obtained result $P(\Delta\Delta) = 0.3\,\%$ agrees well with the experimental upper bound $P(\Delta\Delta) < 0.4\,\%$ [7] and other theoretical estimates [4].

As has been shown in [3], the NNJL model describes well the low-energy nuclear forces for electromagnetic and weak nuclear reactions with the deuteron of astrophysical interest such as the neutron-proton radiative capture $\mathrm{n} + \mathrm{p} \to \mathrm{D} + \gamma$, the solar proton burning $\mathrm{p} + \mathrm{p} \to \mathrm{D} + \mathrm{e}^+ + \nu_{\rm e}$, the pep-process $\mathrm{p} + \mathrm{e}^- + \mathrm{p} \to \mathrm{D} + \nu_{\rm e}$ and reactions of the disintegration of the deuteron by neutrinos and antineutrinos caused by charged $\nu_{\rm e} + \mathrm{D} \to \mathrm{e}^- + \mathrm{p} + \mathrm{p}$, $\bar{\nu}_{\rm e} + \mathrm{D} \to \mathrm{e}^+ + \mathrm{n} + \mathrm{n}$ and neutral $\nu_{\rm e}(\bar{\nu}_{\rm e}) + \mathrm{D} \to \nu_{\rm e}(\bar{\nu}_{\rm e}) + \mathrm{n} + \mathrm{p}$ weak currents.

The important problem which has not been jet clarified in the NNJL model is related to the value of the contribution of the D-wave state to the wave function of the deuteron. In this paper we fill this blank. In sect. 2 we calculate the contribution of the D-wave state to the wave function of the deuteron. We use a relativistically covariant partial-wave analysis developed by Anisovich *et al.* [8] for the description of nucleon-nucleon scattering. We obtain the fraction of the D-wave state of the deuteron wave function relative to the S-wave one equal to $\eta_{\rm d} = 0.0238$. This agrees well with the value $\eta_{\rm d} = 0.0256 \pm 0.0004$

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quoted by Kamionkowski and Bahcall [9][1] who used this parameter for the phenomenological description of the realistic wave function of the deuteron in connection with the calculation of the astrophysical factor $S_{\rm pp}(0)$ for the solar proton burning $\rm p + p \to D + e^+ + \nu_e$ in the potential model approach. In the Conclusion we discuss the obtained result.

2 The D-wave state component of the deuteron

We would carry out the calculation of the value of the D-wave state contribution to the wave function of the deuteron in terms of the amplitude of the transition $\rm n + p \to D$. We show that the neutron-proton pair couples to the deuteron in both the S-wave state and the D-wave state with the fraction of the D-wave state agreeing with low-energy nuclear phenomenology.

In the NNJL model the phenomenological Lagrangian of the npD interaction is defined by [1]

$$\begin{aligned}\mathcal{L}_{\rm npD}(x) = & -ig_{\rm V}[\bar{p^c}(x)\gamma^{\mu}n(x) - \bar{n^c}(x)\gamma^{\mu}p(x)]D^{\dagger}_{\mu}(x)\\ & + \frac{g_{\rm T}}{2M_{\rm N}}[\bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x)]D^{\dagger}_{\mu\nu}(x)\\ & + {\rm h.c.}\,,\end{aligned} \tag{2.1}$$

where $D^{\dagger}_{\mu}(x)$, $n(x)$ and $p(x)$ are the interpolating fields of the deuteron, the neutron and the proton, $D^{\dagger}_{\mu\nu}(x) = \partial_{\mu}D^{\dagger}_{\nu}(x) - \partial_{\nu}D^{\dagger}_{\mu}(x)$ is the deuteron field strength. The phenomenological coupling constant $g_{\rm V}$ is related to the electric quadrupole moment of the deuteron $Q_{\rm D} = 0.286\,{\rm fm}^2$, $g^2_{\rm V} = 2\pi^2 Q_{\rm D} M^2_{\rm N}$ [1], where $M_{\rm N} = 940\,{\rm MeV}$ is the nucleon mass. The coupling constants $g_{\rm V}$ and $g_{\rm T}$ are connected by the relation [1]

$$g_{\rm T} = \sqrt{\frac{3}{8}}\,g_{\rm V}\,, \tag{2.2}$$

which is valid at leading order in the large $N_{\rm C}$ expansion [1].

The amplitude of the transition $\rm n + p \to D$ is determined by

$$\begin{aligned}&\langle k_{\rm D}, \lambda_{\rm D}|\mathcal{L}_{\rm npD}(0)|k_{\rm p}, \sigma_{\rm p}; k_{\rm n}, \sigma_{\rm n}\rangle = \\ &\quad \frac{M({\rm n}({\rm k_n}, \sigma_{\rm n}) + {\rm p}({\rm k_p}, \sigma_{\rm p}) \to {\rm D}(k_{\rm D}, \lambda_{\rm D}))}{\sqrt{2E_{\rm D}V 2E_{\rm n}V 2E_{\rm p}V}}\,,\end{aligned} \tag{2.3}$$

where $(E_{\rm D}, k_{\rm D}, \lambda_{\rm D})$, $(E_{\rm p}, k_{\rm p}, \sigma_{\rm p})$ and $(E_{\rm n}, k_{\rm n}, \sigma_{\rm n})$ are energies, 4-momenta and polarizations of the deuteron, the proton and the neutron, respectively, V is a normalization spatial volume. The wave functions of the initial and final states of the transition $\rm n + p \to D$ are given by

$$\begin{aligned}|k_{\rm p}, \sigma_{\rm p}; k_{\rm n}, \sigma_{\rm n}\rangle &= a^{\dagger}_{\rm p}(k_{\rm p}, \sigma_{\rm p})\, a^{\dagger}_{\rm n}(k_{\rm n}, \sigma_{\rm n})|0\rangle,\\ \langle k_{\rm D}, \lambda_{\rm D}| &= \langle 0|a_{\rm D}(k_{\rm D}, \lambda_{\rm D}),\end{aligned} \tag{2.4}$$

[1] The value $\eta_{\rm d} = 0.0256 \pm 0.0004$ was taken by Kamionkowski and Bahcall from ref. [10].

where $a^{\dagger}_{\rm p}(k_{\rm p}, \sigma_{\rm p})$ and $a^{\dagger}_{\rm n}(k_{\rm n}, \sigma_{\rm n})$ are creation operators of the proton and the neutron, $a_{\rm D}(k_{\rm D}, \lambda_{\rm D})$ is the annihilation operator of the deuteron and $|0\rangle$ is a vacuum wave function. The relativistically invariant amplitude $M({\rm n}({\rm k_n}, \sigma_{\rm n}) + {\rm p}({\rm k_p}, \sigma_{\rm p}) \to {\rm D}(k_{\rm D}, \lambda_{\rm D}))$ reads

$$\begin{aligned}&M({\rm n}({\rm k_n}, \sigma_{\rm n}) + {\rm p}({\rm k_p}, \sigma_{\rm p}) \to {\rm D}(k_{\rm D}, \lambda_{\rm D})) = e^{*\,\nu}(k_{\rm D}, \lambda_{\rm D})\\ &\quad \times \Big\{2ig_{\rm V}[\bar{u^c}(k_{\rm n}, \sigma_{\rm n})\gamma_{\nu}u(k_{\rm p}, \sigma_{\rm p})]\\ &\quad - \frac{2ig_{\rm T}}{M_{\rm N}}[\bar{u^c}(k_{\rm n}, \sigma_{\rm n})\sigma_{\mu\nu}u(k_{\rm p}, \sigma_{\rm p})]\,(k_{\rm n} + k_{\rm p})^{\mu}\Big\},\end{aligned} \tag{2.5}$$

where $\bar{u^c}(k_{\rm n}, \sigma_{\rm n})$ and $u(k_{\rm p}, \sigma_{\rm p})$ are the bispinorial wave functions of the neutron and the proton with 4-momenta $k_{\rm n}$, $k_{\rm p}$ and polarizations $\sigma_{\rm n}$, $\sigma_{\rm p}$; $e^{*\,\nu}(k_{\rm D}, \lambda_{\rm D})$ is a 4-vector of the polarization of the deuteron with 4-momentum $k_{\rm D}$ and polarization $\lambda_{\rm D}$. The 4-momenta $k_{\rm D}$, $k_{\rm n}$ and $k_{\rm p}$ are related by $k_{\rm D} = k_{\rm n} + k_{\rm p}$ due to conservation of energy and momentum.

As has been shown by Anisovich *et al.* [8], the neutron-proton densities describing the S- and D-wave states of a neutron-proton pair are equal to

$$\begin{aligned}\Psi_{\nu}(^3S_1; \sigma_{\rm n}, \sigma_{\rm p}) &= [\bar{u^c}(k_{\rm n}, \sigma_{\rm n})\mathcal{S}_{\nu}u(k_{\rm p}, \sigma_{\rm p})]\,,\\ \Psi_{\nu}(^3D_1; \sigma_{\rm n}, \sigma_{\rm p}) &= [\bar{u^c}(k_{\rm n}, \sigma_{\rm n})\mathcal{D}_{\nu}u(k_{\rm p}, \sigma_{\rm p})]\,,\end{aligned} \tag{2.6}$$

where $\mathcal{S}_{\nu}$ and $\mathcal{D}_{\nu}$ are relativistically covariant operators of the projection onto the S-wave and the D-wave state, respectively [8]:

$$\begin{aligned}\mathcal{S}_{\nu} &= \frac{1}{\sqrt{2s}}\left[\gamma^{\perp}_{\nu} - \frac{2k_{\nu}}{2M_{\rm N} + \sqrt{s}}\right],\\ \mathcal{D}_{\nu} &= \frac{2}{s^{3/2}}\left[\frac{1}{4}\,(4M^2_{\rm N} - s)\gamma^{\perp}_{\nu} - (M_{\rm N} + \sqrt{s})\,k_{\nu}\right].\end{aligned} \tag{2.7}$$

Here $P = k_{\rm p} + k_{\rm n}$, $k = \frac{1}{2}\,(k_{\rm p} - k_{\rm n})$, $s = P^2$, $P \cdot k = 0$ and

$$\gamma^{\perp}_{\nu} = \gamma_{\nu} - \hat{P}\,\frac{P_{\nu}}{s}. \tag{2.8}$$

The neutron-proton densities equations (2.6) are normalized by the condition [8]

$$\begin{aligned}&\frac{1}{3}\int {\rm tr}\{L_{\mu}(\hat{k}_{\rm p} + M_{\rm N})L^{\mu}(-\hat{k}_{\rm n} + M_{\rm N})\}(2\pi)^4\delta^{(4)}(P - k_{\rm p} - k_{\rm n})\\ &\times \frac{{\rm d}^3k_{\rm p}}{(2\pi)^3 2E_{\rm p}}\,\frac{{\rm d}^3k_{\rm n}}{(2\pi)^3 2E_{\rm n}} = \rho_L(s)\,,\end{aligned} \tag{2.9}$$

where $L_{\mu} = \mathcal{S}_{\mu}$ or $\mathcal{D}_{\mu}$, the factor 3 in the denominator of the l.h.s. describes the number of the states of a neutron-proton density with a total momentum $J = 1$, $2J + 1 = 3$, and $\rho_{\mathcal{S}}(s)$ and $\rho_{\mathcal{D}}(s)$ amount to

$$\begin{aligned}\rho_{\mathcal{S}}(s) &= \frac{1}{8\pi}\left(\frac{s - 4M^2_{\rm N}}{s}\right)^{1/2},\\ \rho_{\mathcal{D}}(s) &= \frac{1}{8\pi}\left(\frac{s - 4M^2_{\rm N}}{s}\right)^{5/2}.\end{aligned} \tag{2.10}$$

In the center-of-mass frame of the neutron-proton pair the densities equations (2.6) are equal to

$$\begin{aligned}
&\Psi_0({}^3S_1;\sigma_{\rm n},\sigma_{\rm p}) = [\bar{u^c}(k_{\rm n},\sigma_{\rm n})\mathcal{S}_0 u(k_{\rm p},\sigma_{\rm p})] = 0\,,\\
&\vec{\Psi}({}^3S_1;\sigma_{\rm n},\sigma_{\rm p}) = [\bar{u^c}(k_{\rm n},\sigma_{\rm n})\vec{\mathcal{S}} u(k_{\rm p},\sigma_{\rm p})] =\\
&\quad\frac{1}{\sqrt{2}}\,\varphi^\dagger_{\rm n}(\sigma_{\rm n})\vec{\sigma}\varphi_{\rm p}(\sigma_{\rm p})\,,\\
&\Psi_0({}^3D_1;\sigma_{\rm n},\sigma_{\rm p}) = [\bar{u^c}(k_{\rm n},\sigma_{\rm n})\mathcal{D}_0 u(k_{\rm p},\sigma_{\rm p})] = 0\,,\\
&\vec{\Psi}({}^3D_1;\sigma_{\rm n},\sigma_{\rm p}) = [\bar{u^c}(k_{\rm n},\sigma_{\rm n})\vec{\mathcal{D}} u(k_{\rm p},\sigma_{\rm p})] =\\
&\quad-\frac{1}{2}\,\varphi^\dagger_{\rm n}(\sigma_{\rm n})[3\,(\vec{\sigma}\cdot\vec{v})\,\vec{v}-\vec{v}^{\,2}\,\vec{\sigma}]\,\varphi_{\rm p}(\sigma_{\rm p})\,,
\end{aligned}\tag{2.11}$$

where $\vec{v} = \vec{k}/\sqrt{\vec{k}^{\,2}+M^2_{\rm N}}$ and $\vec{k}$ are the relative velocity and 3-momentum of the neutron-proton pair, $\varphi_{\rm n}(\sigma_{\rm n})$ and $\varphi_{\rm p}(\sigma_{\rm p})$ are spinorial wave functions of the neutron and the proton, respectively. It is obvious that the densities equations (2.11) describe the neutron-proton pair in the S- and D-wave states with a total spin $S = 1$ and a total momentum $J = 1$.

The neutron-proton densities equations (2.11) are normalized by

$$\begin{aligned}
&\frac{1}{3}\sum_{\sigma_{\rm n}=\pm1/2}\sum_{\sigma_{\rm p}=\pm1/2}\vec{\Psi}^\dagger({}^3S_1;\sigma_{\rm n},\sigma_{\rm p})\cdot\,\vec{\Psi}({}^3S_1;\sigma_{\rm n},\sigma_{\rm p}) = 1\,,\\
&\frac{1}{3}\sum_{\sigma_{\rm n}=\pm1/2}\sum_{\sigma_{\rm p}=\pm1/2}\vec{\Psi}^\dagger({}^3D_1;\sigma_{\rm n},\sigma_{\rm p})\cdot\,\vec{\Psi}({}^3D_1;\sigma_{\rm n},\sigma_{\rm p}) =\\
&\quad v^{\,4} = \left(1-\frac{4M^2_{\rm N}}{s}\right)^2.
\end{aligned}\tag{2.12}$$

We would carry out the decomposition of the neutron-proton densities in the amplitude equation (2.5) into the densities with a certain orbital momentum at leading order in the large $N_{\rm C}$ expansion [1–3]. This would allow to consider the neutron and the proton as free particles obeying free equations of motion

$$\begin{aligned}
&\bar{u^c}(k_{\rm n},\sigma_{\rm n})(\hat{k}_{\rm n}+M_{\rm N}) = 0\,,\\
&(\hat{k}_{\rm p}-M_{\rm N})\,u(k_{\rm p},\sigma_{\rm p}) = 0\,.
\end{aligned}\tag{2.13}$$

In order to express the neutron-proton densities in the amplitude equation (2.5) in terms of the projection operators equations (2.7), first we have to exclude the term containing $\sigma_{\mu\nu}$. This can be carried out by using Gordon's identity

$$\begin{aligned}
&[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\sigma_{\mu\nu}u(k_{\rm p},\sigma_{\rm p})]\,\frac{(k_{\rm n}+k_{\rm p})^\mu}{2M_{\rm N}} =\\
&-[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\gamma_\nu u(k_{\rm p},\sigma_{\rm p})]+\frac{k_\nu}{M_{\rm N}}[\bar{u^c}(k_{\rm n},\sigma_{\rm n})u(k_{\rm p},\sigma_{\rm p})].
\end{aligned}\tag{2.14}$$

Substituting eq. (2.14) in eq. (2.5) we get

$$\begin{aligned}
&M({\rm n}({\rm k}_{\rm n},\sigma_{\rm n})+{\rm p}({\rm k}_{\rm p},\sigma_{\rm p})\to{\rm D}(k_{\rm D},\lambda_{\rm D})) =\\
&\quad 2i(g_{\rm V}+2g_{\rm T})\,e^{*\,\nu}(k_{\rm D},\lambda_{\rm D})\\
&\quad\times\Big\{[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\gamma_\nu u(k_{\rm p},\sigma_{\rm p})]\\
&\quad-\frac{2g_{\rm T}}{g_{\rm V}+2g_{\rm T}}\,\frac{k_\nu}{M_{\rm N}}\,[\bar{u^c}(k_{\rm n},\sigma_{\rm n})u(k_{\rm p},\sigma_{\rm p})]\Big\}.
\end{aligned}\tag{2.15}$$

In terms of $\mathcal{S}_\nu$ and $\mathcal{D}_\nu$ vectors, $\gamma^\perp_\nu$ and k_ν are determined by

$$\begin{aligned}
\gamma^\perp_\nu &= \frac{2\sqrt{2}}{3}\,(M_{\rm N}+\sqrt{s})\,\mathcal{S}_\nu-\frac{2}{3}\,\frac{s}{2M_{\rm N}+\sqrt{s}}\,\mathcal{D}_\nu\,,\\
k_\nu &= \frac{1}{3\sqrt{2}}\,(4M^2_{\rm N}-s)\,\mathcal{S}_\nu-\frac{1}{3}\,s\,\mathcal{D}_\nu\,.
\end{aligned}\tag{2.16}$$

Substituting eq. (2.16) into eq. (2.15) and taking into account that $[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\hat{P}u(k_{\rm p},\sigma_{\rm p})] = 0$, we obtain

$$\begin{aligned}
&M({\rm n}({\rm k}_{\rm n},\sigma_{\rm n})+{\rm p}({\rm k}_{\rm p},\sigma_{\rm p})\to{\rm D}(k_{\rm D},\lambda_{\rm D})) =\\
&\quad 4\sqrt{2}i(g_{\rm V}+2g_{\rm T})\,M_{\rm N}\,e^{*\,\nu}(k_{\rm D},\lambda_{\rm D})\\
&\quad\times\{[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\mathcal{S}_\nu u(k_{\rm p},\sigma_{\rm p})]\\
&\quad+\eta_{\rm d}\,[\bar{u^c}(k_{\rm n},\sigma_{\rm n})\mathcal{D}_\nu u(k_{\rm p},\sigma_{\rm p})]\} =\\
&\quad 4\sqrt{2}i(g_{\rm V}+2g_{\rm T})\,M_{\rm N}\,e^{*\,\nu}(k_{\rm D},\lambda_{\rm D})\\
&\quad[\Psi_\nu({}^3S_1;\sigma_{\rm n},\sigma_{\rm p})+\eta_{\rm d}\,\Psi_\nu({}^3{\rm D}_1;\sigma_{\rm n},\sigma_{\rm p})]\,,
\end{aligned}\tag{2.17}$$

where $\eta_{\rm d}$ describes the fraction of the D-wave state in the wave function of the deuteron. It is equal to

$$\eta_{\rm d} = \frac{1}{3\sqrt{2}}\,\frac{2\,g_{\rm T}-g_{\rm V}}{2\,g_{\rm T}+g_{\rm V}}.\tag{2.18}$$

For the derivation of eqs. (2.17) and (2.18) we have set $s = M^2_{\rm D}$ and neglected the contribution of the binding energy of the deuteron in comparison with a nucleon mass $M_{\rm N}$. This means that $4M^2_{\rm N}-s = 0$ when compared with $M^2_{\rm N}$.

Using the relation equation (2.2), the parameter $\eta_{\rm d}$ takes the value

$$\eta_{\rm d} = \frac{1}{3\sqrt{2}}\,\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 0.0238\,.\tag{2.19}$$

This agrees well with the value $\eta_{\rm d} = 0.0256\pm0.0004$ that was used in low-energy nuclear phenomenology for the description of the realistic wave function of the deuteron within the potential model approach [9,10].

3 Conclusion

We have shown that the NNJL model describes well in agreement with low-energy nuclear phenomenology [10] such a fine structure of the deuteron as a contribution of the D-wave state. We have carried out the calculation of the fraction of the D-wave state to the wave function of the deuteron at leading order in the large $N_{\rm C}$ expansion [1].

This has allowed to treat the neutron and the proton as free particles on-mass shell [1–3] obeying free equations of motion. To the decomposition of the amplitude of the transition n + p $\rightarrow$ D into the neutron-proton quantum field configurations having certain orbital momenta and corresponding to the S- and D-wave states, respectively, we have applied a relativistically covariant partial-wave analysis invented by Anisovich *et al.* [8] for the description of nucleon-nucleon scattering with nucleon-nucleon pairs coupled in the states with certain orbital momenta.

The theoretical value of the D-wave state fraction in the wave function of the deuteron $\eta_{\rm d} = 0.0238$ calculated in the NNJL model agrees well with low-energy nuclear phenomenology giving $\eta_{\rm d} = 0.0256 \pm 0.0004$ [10]. The former was quoted by Kamionkowski and Bahcall [9] for the parameterization of the realistic wave function of the deuteron in connection with the calculation of the astrophysical factor $S_{\rm pp}(0)$ for the solar proton burning p + p $\rightarrow$ D + e^+ + $\nu_{\rm e}$. The calculation of the contribution of the D-wave state fraction of the wave function of the deuteron in agreement with low-energy nuclear phenomenology testifies that the NNJL model describes to full extent low-energy tensor nuclear forces playing an important role in low-energy nuclear physics on the whole and for the existence of the deuteron in particular [11].

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